15 Differentiation

Calculator Free

1. [12 marks: 3, 3, 3, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = 5^x$

(b) $y = \sin^5(1 - \sqrt{x})$

(c) $y = \int_0^{e^{2x}} ln(1-x^2)\, dx$

(d) $y = \frac{1-x}{\tan 3x}$

Calculator Free

2. [10 marks: 1, 3, 3, 3]

Find $\frac{dy}{dt}$ for each of the following. You do not need to simplify your answer.

(a) $y = \pi^3$

(b) $y = \tan^3(\pi t^4)$

(c) $y = \int_1^{t^3} \sin^5 2x \, dx + t \int_0^1 5 \, dt$

(d) $y = \frac{\cos(2 - e^{2t})}{t}$

Calculator Free

3. [8 marks: 1, 2, 2, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = \tan(60^\circ)$

(b) $y = \tan(1 - \sqrt{x})$

(c) $y = \int_0^{\pi x} 1 + \cos^4(t)\, dt$

(d) $y = x^2 \ln(\sin 2x)$

Calculator Free

4. [9 marks: 1, 3, 2, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = ln\ e^{2x}$

(b) $y = \cos^3 (3 + \frac{1}{x})$

(c) $y = \int_0^{\tan x} e^{1+t^2}\ dt$

(d) $y = e^{\sin x} \cos x$

Calculator Free

5. [9 marks: 1, 3, 2, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = \ln 2^x$

(b) $y = \sin^5(1 + \ln x)$

(c) $y = \int_0^{x^2} \tan(1+2t)\, dt$

(d) $y = (1 + x^2) \ln \sqrt{x+1}$

Calculator Free

6. [12 marks: 2, 3, 4, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = \sqrt{3x}$

(b) $y = e^{\tan(1-2x)}$

(c) $y = \sin^2(2x)\cos^3(1-x)$

(d) $y = \frac{\sin(2x)}{ln\cos(3x)}$

Calculator Free

7. [10 marks: 1, 3, 3, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = \frac{1}{(1+e)^2}$

(b) $y = \frac{ln(1+\sin x)}{x}$

(c) $y = e^{\tan(\frac{\pi x}{4})} \cos(\frac{\pi x}{4})$

(d) $y = \frac{\sin^2(\pi x)}{\cos(1+x)}$

Calculator Assumed

8. [10 marks: 3, 4, 3]

Find $\frac{dy}{dx}$ in terms of x, for each of the following.

(a) $x = t^2$ and $y = e^{t^3}$

(b) $x = \cos 2\theta$ and $y = \sin 2\theta$

(c) $x = 1 + t$ and $y = \frac{1-t}{1+t}$

Calculator Assumed

9. [11 marks: 3, 4, 4]

Find $\frac{dy}{dx}$ in terms of x, for each of the following.

(a) $x = t^2$ and $y = \ln(1 - t)$

(b) $x = 1 + \cos\theta$ and $y = 2 - \sin\theta$

(c) $x = \frac{1-t^2}{1+t^2}$ and $y = 1 + t$

Calculator Assumed

10. [11 marks: 3, 4, 4]

Find $\frac{dy}{dx}$ in terms of x, for each of the following.

(a) $x = e^{2t}$ and $y = ln(1 + t)$

(b) $x = 1 - 3\sin\theta$ and $y = 3 + 4\cos\theta$

(c) $x = \frac{1-2t}{1+2t}$ and $y = \frac{t^2}{1+2t}$.

15 Differentiation

Calculator Free

1. [12 marks: 3, 3, 3, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = 5^x$

$y = 5^x = e^{\ln 5^x} = e^{x \ln 5}$ ✓

$\Rightarrow \frac{dy}{dx} = \ln 5 \times e^{x \ln 5}$ ✓

$= 5^x \ln 5$ ✓

(b) $y = \sin^5(1 - \sqrt{x})$

$\frac{dy}{dx} = 5 \times [\sin^4(1-\sqrt{x})] \times [\cos(1-\sqrt{x})] \times \left(-\frac{1}{2\sqrt{x}}\right)$

✓ ✓ ✓

(c) $y = \int_0^{e^{2x}} \ln(1-x^2)\, dx$

$\frac{dy}{dx} = 2e^{2x} \ln\left[1 - \left(e^{2x}\right)^2\right]$

✓ ✓ ✓

(d) $y = \frac{1-x}{\tan 3x}$

$\frac{dy}{dx} = \frac{(-1) \times \tan 3x - (1-x)(\sec^2 3x) \times 3}{(\tan 3x)^2}$

✓ ✓ ✓

Calculator Free

2. [10 marks: 1, 3, 3, 3]

Find $\frac{dy}{dt}$ for each of the following. You do not need to simplify your answer.

(a) $y = \pi^3$

$\frac{dy}{dt} = 0$ ✓

(b) $y = \tan^3(\pi t^4)$

$\frac{dy}{dt} = 3 \times [\tan^2(\pi t^4)] \times [\sec^2(\pi t^4)] \times 4\pi t^3$

✓ ✓ ✓

(c) $y = \int_1^{t^3} \sin^5 2x\, dx + t \int_0^1 5\, dt$

$\frac{dy}{dt} = [\sin^5(2t^3)] \times 3t^2 + 5$

✓ ✓ ✓

(d) $y = \frac{\cos(2 - e^{2t})}{t}$

$\frac{dy}{dt} = \frac{(t) \times [-\sin(2-e^{2t})] \times (-2e^{2t}) - [\cos(2-e^{2t})] \times 1}{(t)^2}$

✓ ✓ ✓

Calculator Free

3. [8 marks: 1, 2, 2, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = \tan(60^0)$

$\frac{dy}{dx} = 0$ ✓

(b) $y = \tan(1 - \sqrt{x})$

$\frac{dy}{dx} = [\sec^2(1 - \sqrt{x})] \times \left(-\frac{1}{2\sqrt{x}}\right)$ ✓ ✓

(c) $y = \int_0^{\pi x} 1 + \cos^4(t)\, dt$

$\frac{dy}{dx} = [1 + \cos^4(\pi x)] \times \pi$ ✓ ✓

(d) $y = x^2 \ln(\sin 2x)$

$\frac{dy}{dx} = 2x \ln(\sin 2x) + x^2 \times \frac{2\cos 2x}{\sin 2x}$ ✓ ✓ ✓

Calculator Free

4. [9 marks: 1, 3, 2, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = \ln e^{2x}$

$y = 2x$

$\frac{dy}{dx} = 2$ ✓

(b) $y = \cos^3(3 + \frac{1}{x})$

$\frac{dy}{dx} = 3[\cos^2(3 + \frac{1}{x})] \times [-\sin(3 + \frac{1}{x})] \times -\frac{1}{x^2}$ ✓ ✓ ✓

(c) $y = \int_0^{\tan x} e^{1+t^2}\, dt$

$\frac{dy}{dx} = e^{1+\tan^2 x} \times \sec^2 x$ ✓ ✓

(d) $y = e^{\sin x} \cos x$

$\frac{dy}{dx} = e^{\sin x} \times [-\sin x] + [\cos x \times e^{\sin x}] \times \cos x$ ✓ ✓ ✓

Calculator Free

5. [9 marks: 1, 3, 2, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = \ln 2^x$

$y = x \ln 2$

$\frac{dy}{dx} = \ln 2$ ✓

(b) $y = \sin^5(1 + \ln x)$

$\frac{dy}{dx} = 5[\sin^4(1+\ln x)] \times [\cos(1+\ln x)] \times \frac{1}{x}$

✓ ✓ ✓

(c) $y = \int_0^{x^2} \tan(1+2t)\, dt$

$\frac{dy}{dx} = \tan(1+2x^2) \times 2x$

✓ ✓

(d) $y = (1+x^2)\ln\sqrt{x+1}$

$\frac{dy}{dx} = 2x \ln\sqrt{x+1} + (1+x^2) \times \frac{1}{2(x+1)}$ ✓

✓ ✓

Calculator Free

6. [12 marks: 2, 3, 4, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = \sqrt{3x}$

$y = (\sqrt{3})\sqrt{x}$

$\frac{dy}{dx} = \frac{\sqrt{3}}{2\sqrt{x}}$ ✓✓

(b) $y = e^{\tan(1-2x)}$

$\frac{dy}{dx} = \sec^2(1-2x) \times (-2) \times e^{\tan(1-2x)}$

✓ ✓ ✓

(c) $y = \sin^2(2x)\cos^3(1-x)$

$\frac{dy}{dx} = [2\sin(2x) \times \cos(2x) \times 2] \times \cos^3(1-x) + \sin^2(2x) \times [3\cos^2(1-x) \times -\sin(1-x)] \times -1]$

✓ ✓ ✓ ✓

(d) $y = \frac{\sin(2x)}{\ln\cos(3x)}$

✓ ✓

$\frac{dy}{dx} = \frac{[2\cos(2x)] \times \ln\cos(3x) - \sin(2x) \times \frac{-3\sin(3x)}{\cos(3x)}}{[\ln\cos(3x)]^2}$ ✓

Calculator Free

7. [10 marks: 1, 3, 3, 3]

Find $\frac{dy}{dx}$ for each of the following. You do not need to simplify your answer.

(a) $y = \frac{1}{(1+e)^2}$

$\frac{dy}{dx} = 0$ ✓

(b) $y = \frac{ln(1+\sin x)}{x}$

✓ ✓

$\frac{dy}{dx} = \frac{x \times \frac{\cos x}{1+\sin x} - ln(1+\sin x)}{x^2}$ ✓

(c) $y = e^{\tan(\frac{\pi x}{4})} \cos(\frac{\pi x}{4})$

✓ ✓

$\frac{dy}{dx} = e^{\tan(\frac{\pi x}{4})} \times \frac{\pi}{4} \times -\sin(\frac{\pi x}{4}) + \frac{\pi}{4} \times \sec^2(\frac{\pi x}{4})\, e^{\tan(\frac{\pi x}{4})} \cos(\frac{\pi x}{4})$ ✓

(d) $y = \frac{\sin^2(\pi x)}{\cos(1+x)}$

✓ ✓

$\frac{dy}{dx} = \frac{\cos(1+x)\ 2\pi \sin(\pi x)\cos(\pi x) + \sin(1+x)\ \sin^2(\pi x)}{\cos^2(1+x)}$ ✓

Calculator Assumed

8. [10 marks: 3, 4, 3]

Find $\frac{dy}{dx}$ in terms of x, for each of the following.

(a) $x = t^2$ and $y = e^{t^3}$

$\frac{dx}{dt} = 2t$

$\frac{dy}{dt} = 3t^2 e^{t^3}$ ✓

$\frac{dy}{dx} = \frac{3t^2 e^{t^3}}{2t}$ ✓

$= \frac{3t\, e^{t^3}}{2} = \frac{\pm 3\sqrt{x}\, e^{\pm x^{3/2}}}{2}$ ✓

(b) $x = \cos 2\theta$ and $y = \sin 2\theta$

$\frac{dx}{d\theta} = -2 \sin 2\theta$ ✓

$\frac{dy}{d\theta} = 2 \cos 2\theta$ ✓

$\frac{dy}{dx} = -\frac{x}{y}$ ✓

$= \pm \frac{x}{\sqrt{1-x^2}}$ ✓

(c) $x = 1 + t$ and $y = \frac{1-t}{1+t}$

$\frac{dx}{dt} = 1$

$\frac{dy}{dt} = \frac{-(1+t)-(1-t)}{(1+t)^2} = \frac{-2}{(1+t)^2}$ ✓

$\frac{dy}{dx} = \frac{-2}{(1+t)^2}$ ✓

$= -\frac{2}{x^2}$ ✓

Calculator Assumed

9. [11 marks: 3, 4, 4]

Find $\frac{dy}{dx}$ in terms of x, for each of the following.

(a) $x = t^2$ and $y = \ln(1-t)$

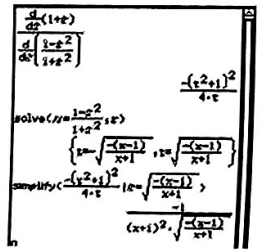
$$\frac{dx}{dt} = 2t$$
$$\frac{dy}{dt} = \frac{-1}{(1-t)} \quad \checkmark$$
$$\frac{dy}{dx} = \frac{-1}{2t(1-t)} \quad \checkmark$$
$$= \frac{\mp 1}{2\sqrt{x}(1 \pm \sqrt{x})} \quad \checkmark$$

(b) $x = 1 + \cos\theta$ and $y = 2 - \sin\theta$

$$\frac{dx}{d\theta} = -\sin\theta \quad \checkmark$$
$$\frac{dy}{d\theta} = -\cos\theta \quad \checkmark$$
$$\frac{dy}{dx} = \frac{x-1}{2-y} \quad \checkmark$$
$$= \frac{x-1}{\pm\sqrt{1-(x-1)^2}} \quad \checkmark$$

(c) $x = \frac{1-t^2}{1+t^2}$ and $y = 1 + t$

$$\frac{dx}{dt} = \frac{-2t(1+t^2) - 2t(1-t^2)}{(1+t^2)^2} = \frac{-4t}{(1+t^2)^2} \quad \checkmark$$
$$\frac{dy}{dt} = 1$$
$$\frac{dy}{dx} = \frac{-(1+t^2)^2}{4t} \quad \checkmark$$
$$= \pm\frac{\sqrt{1+x}}{(1+x)^2\sqrt{1-x}} = \pm\frac{1}{(1+x)^{3/2}\sqrt{1-x}} \quad \checkmark$$



Calculator Assumed

10. [11 marks: 3, 4, 4]

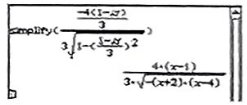
Find $\frac{dy}{dx}$ in terms of x, for each of the following.

(a) $x = e^{2t}$ and $y = \ln(1+t)$

$$\frac{dx}{dt} = 2e^{2t}$$
$$\frac{dy}{dt} = \frac{1}{(1+t)} \quad \checkmark$$
$$\frac{dy}{dx} = \frac{1}{2e^{2t}(1+t)} \quad \checkmark$$
$$= \frac{1}{x(2+\ln x)} \quad \checkmark$$

(b) $x = 1 - 3\sin\theta$ and $y = 3 + 4\cos\theta$

$$\frac{dx}{d\theta} = -3\cos\theta \quad \checkmark$$
$$\frac{dy}{d\theta} = -4\sin\theta \quad \checkmark$$
$$\frac{dy}{dx} = \frac{-4\left(\frac{1-x}{3}\right)}{\pm 3\sqrt{1-\left(\frac{1-x}{3}\right)^2}} \quad \checkmark$$
$$= \pm\frac{4(1-x)}{3\sqrt{(x-2)(4-x)}} \quad \checkmark$$



(c) $x = \frac{1-2t}{1+2t}$ and $y = \frac{t^2}{1+2t}$.

$$\frac{dx}{dt} = \frac{-4}{(1+2t)^2} \quad \checkmark$$
$$\frac{dy}{dt} = \frac{2t^2+2t}{(1+2t)^2} \quad \checkmark$$
$$\frac{dy}{dx} = \frac{-(2t^2+2t)}{4} \quad \checkmark$$
$$= \frac{(x+3)(x-1)}{8(1+x)^2} \quad \checkmark$$